known mechanism, either $\mathrm{Im}\Delta m_{\rho\omega}^2$ may be much larger than $\mathrm{Re}\Delta m_{\rho\omega}^2$, or the nonresonant direct $\omega\rightarrow 2\pi$ process may dominate over the $\omega\rightarrow\rho\rightarrow 2\pi$ decay. The former is compatible with the observed $\omega\rightarrow 2\pi$ rate only if $\mathrm{Re}\Delta m_{\rho\omega}^2$ turns out to be much smaller than a few MeV^2. In the latter case the direct decay simulates the resonant $\omega\rightarrow 2\pi$ decay through an imaginary coupling. Unfortunately, we have no theoretical explanation for such a enormous enhancement in the direct $\omega\rightarrow 2\pi$ decay or the $\rho\rightarrow 3\pi$ decay.

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Eikonal Regge Model for Elastic Scattering Processes*

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The Frautschi-Margolis version of the Regge eikonal model is extended to include secondary Regge trajectories. Physical properties of the model are discussed. In particular, the "shrinkage" of $d\sigma/dt$ observed at present energies (rapid shrinkage for pp and K^+p, little or no shrinkage for $\pi^\pm p$ and K^-p, antishrinkage for $\bar{p}p$) is related to the energy dependence of σ_{tot} (pp and K^+p nearly flat, $\pi^\pm p$ and K^-p falling slowly, $\bar{p}p$ falling rapidly).

I. INTRODUCTION

In recent years it has become increasingly apparent that in order to understand hadron scattering one must consider cuts in the complex angular momentum plane as well as Regge poles.[1] A popular conjecture is to identify the cuts with multiple-scattering terms in the eikonal model.[2] Frautschi and Margolis[3] have aplied this idea to the diffraction peak in elastic scattering, noting the special features which arise when exchange of a Pomeranchuk pole (P) with appreciable slope is supplemented by cuts representing multiple scattering (PP, PPP, ...).

In the present paper we extend the Frautschi-Margolis model, which originally dealt only with the Pomeranchon, to include other Regge poles (R) and their associated cuts (RP, etc.). The formalism of the extension is trivial and follows closely work by Chiu and Finkelstein,[4] but we present it here as a basis for discussion of physical features of the model, and for the detailed fit of the following paper[5] to several elastic reactions at small t.

While the physical nature of the model was already evident in the original paper by Frautschi and Margolis, explicit addition of secondary trajectories clarifies some points, in addition to putting the discussion on a more quantitative basis. The main point we wish to give fresh emphasis has to do with the shrinking (or nonshrinking) of elastic diffraction peaks. Empirically the pp and K^+p forward peaks shrink noticeably, the $\pi^\pm p$ and K^-p peaks exhibit little energy dependence, and the $\bar{p}p$ peak actually broadens with increasing energy. This variable behavior was a well-known embarrassment for the early Regge-pole model. Further variable behavior is found in the total cross section, which is almost flat for pp and K^+p, falls slowly with energy in $\pi^\pm p$ and K^-p scattering, and falls rapidly for $\bar{p}p$. Thus empirically the faster $\sigma_{\mathrm{tot}}(E)$ falls, the less the forward peak shrinks.

We have a natural explanation for this correlation. Multiple-scattering corrections in the eikonal model sharpen the forward peak, by an amount roughly proportional to σ_{tot} [since $\mathrm{Im}A(0)=\sigma_{\mathrm{tot}}/4\pi$ and the double-scattering correction involves the square of the amplitude]. Factors associated with secondary trajectories also act in the same direction, as discussed in Sec. III. This is in accord with the geometrical picture, wherein the width of a diffraction peak is $\Delta\theta\sim\hbar/pR\sim\hbar/p(\sigma_{\mathrm{tot}}/\pi)^{\frac{1}{2}}$. Thus if σ_{tot} *falls* with increasing energy, multiple scattering is *reduced*, the contribution of secondary trajectories is also *reduced*, and the peak tends to *broaden*. The broadening is in competition with the usual Regge shrinkage factor, and the result at present energies is that shrinkage wins when σ_{tot} is flat (pp and K^+p), shrinkage and broadening almost balance when σ_{tot} falls slowly ($\pi^\pm p$ and K^-p), and broadening wins when σ_{tot} falls rapidly ($\bar{p}p$).

Our explanation for the correlation between shrinkage and the energy dependence of σ_{tot} is closely related to the familiar "crossover effect," where if $\sigma_{\mathrm{tot}}(\bar{A}B)$

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[1] See, e.g., the review by J. D. Jackson, Rev. Mod. Phys. **42**, 12 (1970).
[2] R. C. Arnold, Phys. Rev. **140**, B1022 (1965); **153**, 1523 (1967).
[3] S. Frautschi and B. Margolis, Nuovo Cimento **56A**, 1155 (1968); **57A**, 427 (1968); S. Frautschi, O. Kofoed-Hansen, and B. Margolis, *ibid.* **61A**, 41 (1969).
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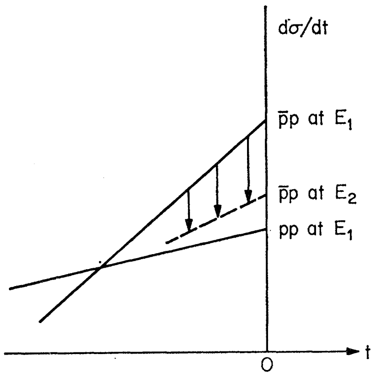

FIG. 1. Crossover effect and broadening of $\bar{p}p$ peak. The solid lines represent $d\sigma/dt$ for reactions $\bar{p}p \to \bar{p}p$ and $pp \to pp$ at energy E_1. We have $\sigma_{\text{tot}}(\bar{p}p) > \sigma_{\text{tot}}(pp)$. As $\sigma_{\text{tot}}(\bar{p}p)$ falls rapidly with increasing energy, $d\sigma/dt$ ($\bar{p}p$) moves to the dashed line at E_2, closer to $d\sigma/dt$ (pp) at E_1 and therefore broader.

$>\sigma_{\text{tot}}(AB)$ then the elastic $d\sigma/dt$ is steeper for $\bar{A}B$ than AB, and the $d\sigma/dt$ *cross* at $t \approx -0.2$ GeV^2. In fact, the broadening associated with a falling $\sigma_{\text{tot}}(\bar{A}B)$ results simply from following the crossover picture through successively higher energies (Fig. 1).

As the non-Pomeranchon Regge exchanges become negligible at high energies, σ_{tot} should stop falling. Therefore, at sufficiently high energies all elastic peaks, including $\bar{p}p \to \bar{p}p$, should shrink.

We now proceed to review the basic equations of the Regge eikonal model in Sec. II, and give the detailed equations for our version of the model in Sec. III. Section III also contains details of how the equations provide the shrinkage pattern described above as well as the other distinctive features noted by Frautschi and Margolis. We point out that the model provides an especially large rise in the total *inelastic* cross section as the energy increases. Finally, in Sec. IV we give a brief discussion.

II. OUTLINE OF MODEL

We consider only the elastic helicity-nonflip amplitude $A(\nu,\mathbf{q})$, which is dominant for elastic scattering at small t, although the model is easily extended to include other amplitudes.[2,3,6,7] In the normalization we use, the cross sections are given by

$$\sigma_{\text{tot}} = 4\pi \,\text{Im} A(\nu,0)\,, \tag{1}$$

$$d\sigma/dt = \pi |A(\nu,\mathbf{q})|^2\,, \tag{2}$$

where $\mathbf{q}$ is the three-momentum transfer ($t=-\mathbf{q}^2$) and ν is the energy variable

$$\nu = \tfrac{1}{2}(s-u)\,. \tag{3}$$

In Glauber's eikonal approximation,[8] the amplitude can be written as

$$A(\nu,\mathbf{q}) = i\int \frac{d^2b}{2\pi}[1-e^{iE(\nu,\mathbf{b})}]e^{i\mathbf{q}\cdot\mathbf{b}}\,, \tag{4}$$

where b is the impact parameter. This formula is valid at high energies and small angles.

By expansion of the exponential in Eq. (4), one obtains

$$A(\nu,\mathbf{q}) = E(\nu,\mathbf{q}) + \tfrac{1}{2}iE\otimes E - \tfrac{1}{6}E\otimes E\otimes E + \cdots\,, \tag{5}$$

where $E=E(\nu,\mathbf{q})$ is the two-dimensional Fourier transform of the "eikonal" $E(\nu,\mathbf{b})$, and the symbol $\otimes$ represents the convolution integral

$$A(\nu,\mathbf{q})\otimes B(\nu,\mathbf{q}) = \int \frac{d^2p}{2\pi} A(\nu,\mathbf{p})B(\nu,\,\mathbf{q}-\mathbf{p})\,. \tag{6}$$

The first term on the right-hand side of Eq. (5) is the "single-scattering term" which corresponds to the Born approximation in nonrelativistic potential scattering[9] and to one-particle exchange in relativistic field theories.[10–12] The further terms in Eq. (5) then correspond to multiple-scattering corrections, two-particle exchanges, etc.

Now the essence of the eikonal Regge model, as first proposed by Arnold,[2] is the assumption that the single-scattering term $E(\nu,\mathbf{q})$ can be equated with the familiar Regge-pole expression for the scattering amplitude:

$$\begin{aligned} E(\nu,\mathbf{q}) &= A_{\text{pole}}(\nu,\mathbf{q}) \\ &= P(\nu,\mathbf{q}) + R(\nu,\mathbf{q})\,, \end{aligned} \tag{7}$$

where $R(\nu,\mathbf{q})$ stands for the sum of all contributing Regge trajectories, and $P(\nu,\mathbf{q})$ corresponds to the diffractive part of the amplitude, usually going under the name of "Pomeranchuk exchange." This is a very reasonable supposition in light of the connection of $E(\nu,\mathbf{q})$ with one-particle exchange terms discussed above. But when it comes to writing an expression for $P(\nu,\mathbf{q})$, the model has taken two different forms.

Version I of the model is a "hybrid form," proposed by Chiu and Finkelstein[4] in which the Pomeranchuk contribution is given an optical form, taken from the

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[9] This observation seems to have been first made by G. Molière, Z. Naturforsch. **2a**, 133 (1947).

[10] F. Englert, P. Nicoletopoulos, R. Brout, and C. Truffin, Nuovo Cimento **64A**, 561 (1969); H. D. I. Abarbanel and C. Itzykson, Phys. Rev. Letters **23**, 53 (1969); R. Torgerson, Phys. Rev. **143**, 1194 (1966). These authors summed a certain set of Feynman ladder diagrams for elastic scattering of two particles interacting by exchange of massive quanta.

[11] S. Chang and S. Ma, Phys. Rev. Letters **22**, 1334 (1969) and H. Cheng and T. T. Wu, Phys. Rev. **186**, 1611 (1969) obtained the eikonal amplitude for a subset of Feynman diagrams in electron-electron scattering.

[12] M. Levy and J. Sucher, Phys. Rev. **186**, 1656 (1969).

work of Chou and Yang[13]; e.g., for pp scattering they write

$$P(\nu,\mathrm{q})=iK[F(t)]^2, \tag{8}$$

where $F(t)$ is essentially the charge form factor of the proton. In Regge language, this corresponds to a fixed Pomeranchuk pole, $\alpha_P(t)\equiv 1$, and implies that $d\sigma/dt$ will approach a fixed, limiting shape at high energies which is typical of a classical, absorptive target. This version of the model has been used to fit pp and $\bar{p}p$ differential cross sections[4,14,15] and πN scattering data,[6] and succeeds in explaining most of the t-dependent features.

Version II of the model was put forward by Frautschi and Margolis,[3] and treats the Pomeranchuk contribution just like any other moving Regge pole. This version is capable of explaining the same features as version I but it differs as regards the energy dependence. For instance, it predicts that the forward diffractive peak in $d\sigma/dt$, rather than approaching a limiting shape, will continue to shrink logarithmically at high energies. The latest data from Serpukhov on elastic pp scattering[16] favor version II. But so far, the only applications of this model have been that of Frautschi and Margolis themselves, who studied qualitative features[17] only and neglected the effect of secondary trajectories, and another by Kaplan and Schiff,[15] who made a semiquantitative fit of pp and $\bar{p}p$ scattering data.

Consequently, in Sec. III we will take a closer look at the many interesting properties of this version of the eikonal model.

III. DETAILS AND SIGNIFICANT FEATURES OF MODEL

In Eq. (7) we expressed the eikonal as the sum of the Pomeranchuk contribution plus those of the secondary Regge trajectories ρ, f, A_2, and ω. (For the purposes of this discussion we will assume complete exchange degeneracy among these trajectories.) For the Pomeranchon we use the simplest possible expression for an even-signature, linear Regge trajectory with intercept $\alpha_P^0=1$ and slope α_P',

$$\alpha_P(t)=\alpha_P^0+\alpha_P' t, \tag{9}$$

giving the amplitude

$$\begin{aligned} P&=iCe^{a_P t}(\nu e^{-i\pi/2})^{\alpha_P(t)-1}\\ &=iCe^{-\gamma q^2}, \end{aligned} \tag{10}$$

where

$$\gamma=a_P+\alpha_P'(\ln\nu-\tfrac{1}{2}i\pi), \tag{11}$$

and a_P contains both the term $-\alpha_p'\ln\nu_0{}^P$ and any exponential t dependence the residue may have. Similarly, for the sum of all degenerate, secondary trajectories,

$$\alpha_R(t)=\alpha_R^0+\alpha_R' t, \tag{12}$$

we write for the amplitude

$$R=(Ae^{-\alpha q^2}-Be^{-i\pi\alpha_R^0}e^{-\beta q^2})\nu^{\alpha_R^0-1}, \tag{13}$$

with

$$\begin{aligned} \alpha&=a_R+\alpha_R'\ln\nu,\\ \beta&=a_R+\alpha_R'(\ln\nu-i\pi). \end{aligned} \tag{14}$$

In coordinate space, the eikonal $E(\nu,\mathbf{b})$ then takes the form

$$E(\nu,\mathbf{b})=\frac{iC}{2\gamma}e^{-b^2/4\gamma}+\left(\frac{A}{2\alpha}e^{-b^2/4\alpha}-\frac{B}{2\beta}e^{-i\pi\alpha_R^0}e^{-b^2/4\beta}\right)\nu^{\alpha_R^0-1}. \tag{15}$$

If this is substituted into Eq. (4) there results an expression for the full scattering amplitude which can be evaluated to any required accuracy and includes all the multiple-scattering corrections as well as the simple pole contributions. By expanding the exponent and integrating over b, it can be written as a double series:

$$A(\nu,\mathrm{q})=-i\sum_{n=1}^{\infty}\sum_{m=0}^{n}A_{nm}(\nu,\mathrm{q}). \tag{16}$$

Here $A_{nm}(\nu,\mathrm{q})$ is the amplitude corresponding to m R-exchanges and $(n-m)$ P-exchanges:

$$A_{nm}(\nu,\mathrm{q})=(\nu^{\alpha_R^0-1})^m\sum_{k=0}^{m}\frac{x(n,m,k)}{(n-m)!(m-k)!k!}\times\left(-\frac{C}{2\gamma}\right)^{n-m}\left(\frac{iA}{2\alpha}\right)^{m-k}\left(-\frac{Be^{i\pi(\frac{1}{2}-\alpha_R^0)}}{2\beta}\right)^{k}\times\exp[-\tfrac{1}{2}q^2x(n,m,k)], \tag{17}$$

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[13] T. T. Chou and C. N. Yang, Phys. Rev. Letters **20**, 1213 (1968); Phys. Rev. **170**, 1591 (1968).

[14] A. Capella, J. Kaplan, A. Krzywicki, and D. Schiff, Nuovo Cimento **63A**, 141 (1969).

[15] J. Kaplan and D. Schiff, Nuovo Cimento Letters **3**, 19 (1970).

[16] G. G. Beznogikh *et al.*, Phys. Letters **30B**, 274 (1969).

[17] We would like to take this opportunity to answer the criticism of Capella *et al.* (Ref. 14), who find that the large-t behavoir of the single-scattering term is very important in their model, whereas Frautschi and Margolis claim it is unimportant in the Frautschi-Margolis model because multiple scattering by a series of small momentum transfers dominates at large t. Actually each group is correctly describing the effects of its *own* choice of $P(\nu,\mathrm{q})$. Capella *et al.* employed the Chou-Yang version. For the choice $[F(t)]^2=(1-at)^{-4}$ one readily verifies that a single scattering with $F^2\to t^{-4}$ (or a double scattering with most of the momentum transfer occurring in just one of the scatterings) eventually dominates a double scattering $\sim F^2(t_1)F^2(t_2)\to(t_1)^{-4}(t_2)^{-4}\sim(\frac{1}{4}t)^{-8}$ in which the momentum transfer $(-t)^{1/2}\leq(-t_1)^{1/2}+(-t_2)^{1/2}$ is shared about equally by both scatterings. The same is true, over the *finite* range of t considered by Capella *et al.*, for the sum of exponentials they design to reproduce the form $(1-at)^{-4}$ at $0\leq|t|<1.5$ GeV^2. By contrast, Frautschi and Margolis use an amplitude with $P(\nu,t)\sim e^{\gamma t}$, and here double scattering with $(-t_1)^{1/2}\approx(-t_2)^{1/2}\approx\frac{1}{2}(-t)^{1/2}$ $[e^{\gamma t_1}e^{\gamma t_2}\approx e^{\gamma t/2}]$ dominates single scattering $(\sim e^{\gamma t})$ at sufficiently large t.

where

$$x(n,m,k)=\left(\frac{n-m}{2\gamma}+\frac{m-k}{2\alpha}+\frac{k}{2\beta}\right)^{-1}. \qquad (18)$$

Note that the higher-order terms exhibit a logarithmic dependence on ν through the factors α, β, and γ, and therefore they correspond to cuts in the angular momentum plane. It has been shown by Chiu and Finkelstein,[4] and it can be checked explicitly from Eq. (17), that these Regge cuts have all the properties generally expected of them. In other words, this model provides an explicit, absorptive-type prescription for calculating the strength of the Regge cuts without introducing any more parameters than a simple pole model.

From the general equation (17), it is easy to pick out the terms corresponding to the Pomeranchuk and single Regge exchange[18] with all their absorptive cuts (due to repeated P exchange):

$$A^{\mathrm{Pom}}(\nu,\mathbf{q})=iC\sum_{n=1}^{\infty}\frac{1}{nn!}\left(-\frac{C}{2\gamma}\right)^{n-1}e^{-(\gamma/n)q^2}, \qquad (19)$$

$$A^{\mathrm{Reg}}(\nu,\mathbf{q})=A\nu^{\alpha_R^0-1}\sum_{n=1}^{\infty}\frac{1}{n!}\left(-\frac{C}{2\gamma}\right)^{n-1}\frac{\alpha_n}{\alpha}e^{-(\alpha_n/n)q}$$

$$-Be^{-i\pi\alpha_R^0}\nu^{\alpha_R^0-1}\sum_{n=1}^{\infty}\frac{1}{n!}\left(-\frac{C}{2\gamma}\right)^{n-1}\frac{\beta_n}{\beta}e^{-(\beta_n/n)q^2}, \qquad (20)$$

where α_n and β_n are given by

$$\frac{n}{\alpha_n}=\frac{1}{\alpha}+\frac{n-1}{\gamma}, \qquad \frac{n}{\beta_n}=\frac{1}{\beta}+\frac{n-1}{\gamma}. \qquad (21)$$

Now let us review the ways in which the multiple-scattering corrections, or Regge-cut terms, affect experimental quantities:

(i) The first two terms in Eq. (19) give an amplitude in the forward directions:

$$A(\nu,0)=iC\left[1-\frac{C}{8[a_P+\alpha_P'(\ln\nu-\frac{1}{2}i\pi)]}\right]. \qquad (22)$$

The imaginary part coming from the first cut (i.e., the second term) has opposite sign to the first term. This means that once the Regge-exchange terms have died away, σ_{tot} will begin to *rise* towards its asymptotic limit $4\pi C$; the cut produces a "shadowing" effect which dies away logarithmically as ν increases.[19] But note that if $\alpha_P'=0$, as in version I of this model, this rise would not occur. Physically, the difference is that in version I the radius and transparency of the target are fixed and the shadowing does not change with energy, whereas in version II the radius expands as $\ln\nu$, the transparency increases correspondingly to maintain a constant σ_{tot}, and the shadowing effect in the increasingly transparent medium dies away.

(ii) Another significant feature of the model can be seen from Eq. (22). The real part of the amplitude coming from the cut is positive. Since the real part coming from the Regge terms is generally negative, but dying away like $\nu^{-1/2}$, we can predict that the ratio $\mathrm{Re}A(0)/\mathrm{Im}A(0)$ will change sign at energies of the order of 10^3–10^4 GeV.

(iii) From Eq. (19) we see that the cuts have a smaller slope than the pole. And since the signs of their imaginary parts alternate, we get dips in $d\sigma/dt$. The first dip occurs approximately when the imaginary parts of the Pomeranchuk pole and its first cut are equal and opposite. The next dip is produced by cancellation between the first and second cut, and so on. These dips are common to all eikonal models for elastic scattering; an additional feature of the Frautschi-Margolis version is that the dips are automatically filled in to some extent by the real parts coming from the P pole and cuts, and thus appear as mere "breaks" at present energies.[20]

Since the slopes of the pole and all the cuts increase with energy, all the dips will move inward to smaller $|t|$ at very high energies. This is again in contrast to version I, which predicts that the differential cross sections will approach a fixed shape. In both versions, the dips should become more prominent at large energies.

As discussed in the quoted references,[4,6,7,14] these diffraction-like dips or breaks are in good agreement with experimental data without favoring any particular version of this model.

(iv) The slope of the forward diffraction peak, b, defined at small t by

$$\frac{d\sigma}{dt}=\left.\frac{d\sigma}{dt}\right|_{t=0}e^{bt}=\pi|A|^2, \qquad (23)$$

can be calculated from the equation

$$b=\frac{A'(0)}{A(0)}+\mathrm{c.c.} \qquad (24)$$

Keeping the terms P, PP, R, and RP, and taking $\alpha_R^0=\frac{1}{2}$ for simplicity, we then find that b can be written as a sum of three distinctive parts,

$$b=b_0+b_-+b_+, \qquad (25)$$

[18] Note: We often refer to "Regge exchange," meaning exchange of the *secondary* Regge poles ρ, f, etc., as distinct from the Pomeranchon.

[19] This and many other features of our model were also obtained by V. N. Gribov and A. A. Migdal, Yadern. Fiz. **8**, 1213 (1968) [Soviet J. Nucl. Phys. **8**, 703 (1969)] on the basis of Reggeon graph techniques.

[20] Filled-in dips, called "oscillations," were also obtained in a noneikonal study of cuts by A. A. Anselm and I. T. Dyatlov, Phys. Letters **24B**, 479 (1967).

where

$$b_0=2(a_P+\alpha_P'\ln\nu)+\tfrac{1}{8}C,$$

$$b_-=\frac{2A}{\nu^{1/2}C}\operatorname{Im}\left\{\alpha-\gamma+\tfrac{1}{2}C\left[\left(\frac{\gamma}{\alpha+\gamma}\right)^2+\frac{\alpha}{4\gamma}-\tfrac{3}{8}\right]\right\}, \tag{26}$$

$$b_+=\frac{2B}{\nu^{1/2}C}\operatorname{Re}\left\{\beta-\gamma+\tfrac{1}{2}C\left[\left(\frac{\gamma}{\beta+\gamma}\right)^2+\frac{\beta}{4\gamma}-\tfrac{3}{8}\right]\right\}.$$

The first part b_0, coming from the Pomeranchon and its first cut, increases for all energies as long as $\alpha_P'\neq 0$, tending to produce "shrinkage" of the forward diffraction peak. The next part b_-, arising from the real part A of the Regge-pole term, will be negative at all energies and cause shrinkage, since A is generally negative. But the last part b_+ is found to be positive in our numerical work, and can cause *antishrinkage* at low energies if it is large enough compared to b_0.[21]

In more detail, the first part of b_+ (proportional to $\beta-\gamma$) reflects the difference in momentum transfer dependence between R exchange ($\sim e^{\beta t}$) and P exchange ($\sim e^{\gamma t}$), while the second part of b_+ [proportional to $(B/C)\times C$] is a cut term representing absorption of the pole exchanges. In the numerical work of the following paper,[5] we find that both parts of b_+ have the same sign and contribute to the crossover effect and antishrinkage. At $t=0$ the first part is relatively more important (as much as 80% of the whole effect) while at larger t the cut terms become comparably important. If one multiplies the cut term by an arbitrary factor greater than 1, as is done by various groups,[22–24] the cut term is important even at $t=0$.

While putting the finishing touches on this paper we received an unpublished report from Barger and Cline,[25] who also take a geometrical viewpoint and consider the relation between falling σ_{tot} and antishrinkage of the $\bar{p}p$ peak. They do not explicitly include cuts, so in their model the antishrinkage is attributed entirely to sharper peaking of secondary Regge-pole exchanges than Pomeranchuk pole exchange. In our language, they say b_+ is positive because $\beta-\gamma>0$, whereas we say it is positive because both $\beta-\gamma$ *and* the cut contributions are positive. The relative numerical influences have been described above; evidently we agree that the influence of $\beta-\gamma$ is very important. Strictly speaking, of course, cuts are also needed to provide the crossover effect.

We can relate b_+ to the physical discussion of the Introduction as follows. First, b_+ arises from the imaginary part B of the Regge-pole term. The presence of B implies an augmented but falling σ_{tot}, as in $\bar{p}p$; its absence implies a smaller but more nearly constant σ_{tot} as in pp. Thus we obtain the crossover effect: $\sigma_{\text{tot}}(\bar{p}p)>\sigma_{\text{tot}}(pp)\rightarrow B(\bar{p}p)>B(pp)\rightarrow$ (unless $\beta-\gamma$ is large and negative) $b_+(\bar{p}p)>b_+(pp)\rightarrow\bar{p}p$ peak narrower than the pp peak. Now as the energy rises and b_+ falls off like $\nu^{-1/2}$, the decline of b_+ removes the narrowing of the $\bar{p}p$ peak, so we get antishrinking. This effect competes with the normal shrinking of b_0.

The numerical validity of the diffractive explanation of "crossover" in $d\sigma/dt\,(\bar{p}p)-d\sigma/dt\,(pp)$ has been demonstrated by Capella *et al.*[14] They obtain a fit to the observed crossover near $t=-0.2$ GeV^2. In simple pole models this crossover should occur at $t=-0.5$ GeV^2, where the ω contribution goes to zero. The only way to move it in to $t=-0.2$ GeV^2 in a pure pole model was to insert an artificial zero in the ω residue at this point, a rather ugly procedure.

Actually both the crossover effect and the expanding $\bar{p}p$ peak are also obtained in version *I* of the Regge eikonal model, with $\alpha_P(t)=1$. But at the highest energies available at Serpukhov the slope of the diffractive peak is almost equal to the Pomeranchuk contribution, b_0, alone, and it is still shrinking.[16] This is the strongest evidence in favor of the Frautschi-Margolis version of the eikonal model, as was pointed out by Kaplan and Schiff.[15]

(v) Recently Grigorov *et al.*[26] have studied the cross section for particle production by cosmic-ray protons on carbon. They find a 20% rise from 10 GeV to 10^3 GeV. This result, though not confirmed by other experimental groups, is precisely what one would expect in the Frautschi-Margolis model.

The cross section for particle production in pp collisions is

$$\sigma_{\text{prod}}=\sigma_{\text{tot}}-\sigma_{\text{el}}. \tag{27}$$

In our model σ_{tot} rises, and can be approximated at extremely high energies with the aid of Eq. (22):

$$\sigma_{\text{tot}}\simeq 4\pi C[1-C/8\alpha_P'\ln\nu]. \tag{28}$$

At the same time the integrated elastic cross section, which is approximately

$$\sigma_{\text{el}}=\int dt\frac{d\sigma}{dt}\simeq\frac{1}{b}\frac{d\sigma}{dt}\bigg|_{t=0}=\frac{\pi}{b}|A(\nu,0)|^2\simeq\frac{\sigma_{\text{tot}}^2}{16\pi b}, \tag{29}$$

[21] In their analysis of this subject, Frautschi and Margolis kept only the b_0 term explicitly, but they let the coupling C have an energy dependence to simulate the effect of secondary trajectories. This led them to the same physical picture as we obtain here [Ref. 3 and S. Frautschi, in *Theory and Phenomenology in Particle Physics*, edited by A. Zichichi (Academic, New York, 1969), p. 710], though of course the phases and t dependence of the Regge terms were treated incorrectly by this method.

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goes to zero as the peak width shrinks. Specifically, from Eqs. (22) and (26), we estimate

$$\sigma_{\text{el}} \simeq \pi C^2/2\alpha_P' \ln\nu. \tag{30}$$

Both the increase of σ_{tot} and the decrease of σ_{el} contribute to an over-all increase of σ_{prod}.

For protons on carbon, there is also the normal Glauber shadowing of the nucleons at the backside of the nucleus by the nucleons in front. In our Reggeized model this shadowing, like the shadowing of a nucleon by its own frontside, decreases logarithmically at high energy. This further source of increase in σ_{prod} has been estimated by Udgaonkar and Gell-Mann.[27] The decrease in nuclear shadowing is found to be somewhat less important than the decrease in nucleon self-shadowing at present energies, because the logarithmically increasing effective radius of the nucleon-nucleon interaction is still less than the nuclear radius.

IV. DISCUSSION

We have already compared version I (fixed Pomeranchon) and version II (moving Pomeranchon) of the Regge eikonal model. There are a number of other related models which begin with a moving Pomeranchuk pole and add one cut term, but with a numerical coefficient not determined by the eikonal and with the higher-order cuts omitted. Barger and Phillips[23] and Lendel and Ter-Martirosyan[24] have recently proposed models of this type. Evidently the qualitative features obtained are similar to the Frautschi-Margolis model. Concerning the differences, we must acknowledge that the numerical strength of the Pomeranchuk cuts may well be different from our value due to virtual diffractive dissociation, as emphasized by the Michigan group.[22] At the same time, however, the higher-order cuts must also be there, and we have thought it desirable to see what one gets by following the Glauber-Arnold prescription for them all the way. As seen in detail in the following paper, this provides models with fewer parameters, and is thus easier to test on data.

In conclusion, our object has been to emphasize the many attractive features of the eikonal Regge model. It ties together in a neat fashion several of the generally accepted approaches to high-energy scattering. First of all, it provides a prescription for calculating the effects of Regge cuts from the presence of Regge poles. Secondly, it exhibits many features one would expect on the basis of diffraction theory; thus it can be expected to approach more closely to unitarity than a simple Regge-pole model. It also can easily accommodate the idea that the hadrons are composite objects, through its use of the eikonal approximation. In the following paper, we attempt a quantitative fit of the model to small-angle elastic scattering data.

[27] B. M. Udgaonkar and M. Gell-Mann, Phys. Rev. Letters 8, 346 (1962).